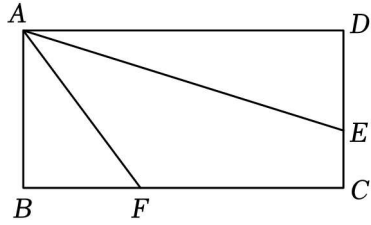


2024 春季初三数学每日一题打卡 017

试题来源：2023 南京市建邺区二模

如图，在矩形 $ABCD$ 中， $AD = 2AB$ ，点 E 在 CD 上， $\angle BAE$ 的平分线交 BC 于点 F ，若 $ED = BF = 3$ ，则 $AE =$ _____.



试题解析:

如图,在矩形 $ABCD$ 中, $AD = 2AB$, 点 E 在 CD 上, $\angle BAE$ 的平分线交 BC 于点 F , 若 $ED = BF = 3$, 则 $AE = \underline{\quad 9 \quad}$.

延长 AF 、 DC 交于点 G , 设 $CD = m$,

\because 四边形 $ABCD$ 是矩形, $AD = 2AB$,

$\therefore AB \parallel CD$, $AB = CD = m$, $BC = AD = 2AB = 2m$, $\angle B = \angle BCD = \angle D = 90^\circ$,

$\therefore \angle GCF = 90^\circ$,

$\because ED = BF = 3$, $\angle GFC = \angle AFB$,

$\therefore CE = m - 3$, $CF = 2m - 3$,

$\because AB \parallel CD$

$\therefore \frac{GC}{AB} = \frac{CF}{BF}$,

$\therefore GC = \frac{AB \cdot CF}{BF} = \frac{m(2m-3)}{3} = \frac{2}{3}m^2 - m$,

$\therefore GE = GC + CE = \frac{2}{3}m^2 - m + m - 3 = \frac{2}{3}m^2 - 3$,

$\because \angle BAE$ 的平分线交 BC 于点 F ,

$\therefore \angle EAF = \angle BAF$,

$\because \angle G = \angle BAF$,

$\therefore \angle EAF = \angle G$,

$\therefore GE = AE$,

$\therefore GE^2 = AE^2$

$\therefore GE^2 = AD^2 + ED^2$,

$\therefore \left(\frac{2}{3}m^2 - 3\right)^2 = (2m)^2 + 3^2$,

整理得 $m^4 - 18m^2 = 0$,

$\therefore m^2(m^2 - 18) = 0$

$\because m \neq 0$,

$\therefore m^2 - 18 = 0$,

解得 $m_1 = 3\sqrt{2}$, $m_2 = -3\sqrt{2}$ (不符合题意, 舍去),

$\therefore AE = GE = \frac{2}{3}m^2 - 3 = \frac{2}{3} \times (3\sqrt{2})^2 - 3 = 9$.

角平分线+平行线构造等腰三角形, 忽然发现, 原来解压轴题如此简单.

