2026春季初三数学每日一题打卡005

如图1，$\triangle ABC$内接于$\odot O$，直径CD交AB于点E，满足$\angle BEC=3\angle ACD$.

(1) 若$\angle BEC=75^\circ$，求$\angle B$的度数.

(2) 求证：$AB=AC$.

(3) 如图2，连接BD. 若$BC=8$，$\tan\angle ABD=\dfrac{1}{2}$，求$\dfrac{DE}{EC}$的值.

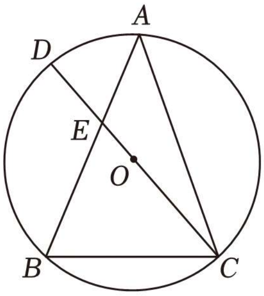

图 1

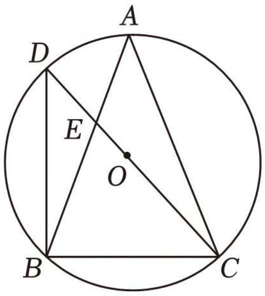

图 2

试题解析

如图1，$\triangle ABC$内接于$\odot O$，直径CD交AB于点E，满足$\angle BEC=3\angle ACD$.

(1) 若$\angle BEC=75°$，求$\angle B$的度数.

(2) 求证：$AB=AC$.

(3) 如图2，连接BD. 若$BC=8$，$\tan\angle ABD=\frac{1}{2}$，求$\frac{DE}{EC}$的值.

(1) 解：$\triangle ABC$内接于$\odot O$，直径CD交AB于点E，满足$\angle BEC=3\angle ACD$，$\angle BEC=75°$，

如图1，连接OA，可得$OA=OC$，$\angle ACD=\angle OAC$，$\therefore\angle ACD=\angle OAC=\frac{\angle BEC}{3}=25°$，

$\therefore\angle AOC=180°-\angle OAC-\angle ACD=130°$，$\therefore\angle B=\frac{1}{2}\angle AOC=65°$；

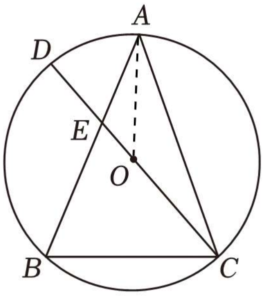

图 1

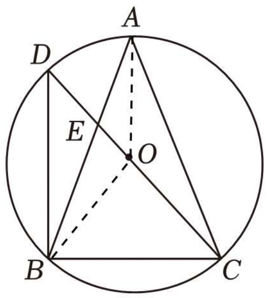

图 2

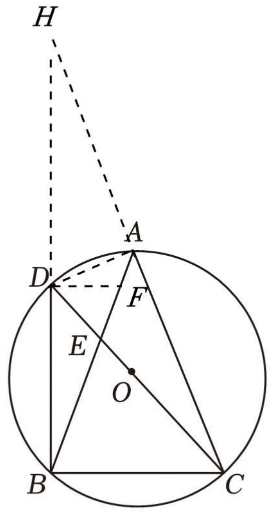

图3

(2) 证明：$\angle BEC=3\angle ACD=\angle ACD+\angle OAC+\angle OAB$，如图2，连接$OA$、$OB$，

可得$OA=OC=OB$，$\angle ACD=\angle OAC$，$\angle OAB=\angle OBA$，$\angle OBC=\angle OCB$，

$\therefore\angle OAB=\angle OBA=\angle ACD$，$\therefore\angle OBA+\angle OBC=\angle OCB+\angle ACD$，即$\angle ABC=\angle ACB$，

$\therefore AB=AC$，$\therefore\triangle ABC$为等腰三角形；

(3) 解：连$\angle BEC=3\angle ACD=\angle BAC+\angle ACD$，如图3，接$AD$，过点$D$，作$DF\parallel BC$交$AB$于点$F$，作$CA$，$BD$的延长线交于点$H$，$\therefore\angle BAC=2\angle ACD$，

$\because\angle BAC=\angle H+\angle ABD$，$\angle ABD=\angle ACD$，$\therefore 2\angle ACD=\angle H+\angle ACD$，

$\therefore\angle H=\angle ACD=\angle ABD$，$\therefore HD=CD$，$AH=AB=AC$，

$\because\tan\angle ABD=\frac{1}{2}$，$\therefore\tan\angle ACD=\tan\angle H=\frac{1}{2}$，

$\because CD$是$\odot O$的直径，$BC=8$，$\therefore\angle DAC=90°=\angle DAH$，$\angle DBC=90°$，

$\therefore\frac{DA}{AC}=\frac{BC}{HB}=\frac{1}{2}$，即$\frac{8}{HB}=\frac{1}{2}$，$\therefore HB=16$，

在直角三角形BCH中，由勾股定理得：$HC=\sqrt{HB^2+BC^2}=8\sqrt{5}$，$\therefore AB=AH=AC=4\sqrt{5}$，

$\therefore AD=\frac{1}{2}AC=2\sqrt{5}$，$\because\angle DAH=90°$，

在直角三角形ADH中，由勾股定理得：$DH=\sqrt{AH^2+AD^2}=10$，

$\therefore BD=BH-DH=16-10=6$，$\because DF\parallel BC$，$\therefore\angle FDB=\angle DBC=90°$，

$\therefore\tan\angle DBF=\frac{DF}{BD}=\frac{1}{2}$，即$\frac{DF}{6}=\frac{1}{2}$，$\therefore DF=3$，

$\because DF\parallel BC$，$\therefore\triangle DFE\backsim\triangle CBE$，$\therefore\frac{DE}{EC}=\frac{DF}{BC}=\frac{3}{8}$.